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Relationships between normal-incidence and statistical absorption coefficients

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RELATIONSHIPS BETWEEN NORMAL-INCIDENCE AND STATISTICAL ABSORPTION COEFFICIENTS

Summary

The theory underlying the absorption of sound at normal incidence is summarised and its extension to include absorption at random incidence is discussed. A chart of the relationships between the normal-incidence and random-incidence or "statistical" absorption coefficients is given, and a method of using this chart to obtain values of the statistical absorption coefficient from impedance tube measurements is described.

Issued under the authority of

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RELATIONSHIPS BETWEEN NORMAL-INCIDENCE AND STATISTICAL ABSORPTION COEFFICIENTS

LIST OF SYMBOLS

A	General complex quantity	
c	Velocity of sound in air	
e	Base of natural logarithms	
f	Frequency	
I_i, I_r	Sound intensity (power) in incident and reflected waves	
j	$\sqrt{-1}$	
k	general constant	
P_i, P_r	Sound pressure in incident and reflected waves	
P_t	overall sound pressure in front of a surface	
R_N	Normal-incidence reflection coefficient $(=\lvert R_N\rvert e^{j\gamma} = R_R + jR_I)$	
R_θ	Oblique-incidence reflection coefficient at angle of incidence θ	
S	Ratio of maximum to minimum sound pressure	
T	Temperature	
t	time variable	
v_i, v_r	air velocity in incident and reflected waves	
v_t	air velocity in front of (and normal to) a surface	
x_{min}	distance of first sound pressure minimum from surface	
Z	acoustic impedance	
α_N	normal-incidence absorption coefficient	
α_{NLim}	Highest possible value of α_N for given value of β	
α_{stat}	random-incidence or statistical absorption coefficient	
α_θ	oblique-incidence absorption coefficient at angle of incidence θ	
β	argument of specific acoustic impedance ζ	
β_{Lim}	highest possible value of β for given value of α_N	
γ	argument of normal-incidence reflection coefficient R_N	
ζ	specific acoustic impedance $(=\lvert\zeta\rvert e^{j\beta} = \zeta_R + j\zeta_I)$	
θ	angle of incidence	
λ	wavelength of sound wave in air	
ρ_0	density of air	
ω	angular frequency	
*	conjugate	of complex quantity
$\lvert\ \rvert$	modulus	
subscript R	real part	
subscript I	imaginary part	
ˆ	peak value	
$(\ )_{rms}$	root-mean-square value	
bar over symbol	time average	
subscripts max, min	Maximum and minimum values (other than in the case of x_{min})	

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RELATIONSHIPS BETWEEN NORMAL-INCIDENCE AND STATISTICAL ABSORPTION COEFFICIENTS

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1. Introduction

Measurement of the sound absorbing properties of materials used in the acoustic treatment of studios is usually carried out using the reverberation room technique, in which a sample of the material under test is introduced into a reverberation room and the resulting change in reverberation time is measured. A disadvantage of this method of measurement is the relatively large amount of material (say 10m^2 or 100ft^2) required, and the alternative "impedance tube" technique[1a, 2a] which involves only a small sample of the material, is therefore sometimes used. This latter technique is subject to certain limitations, one of which is that the quantity that is measured directly is the "normal-incidence absorption coefficient" (α_N), which refers only to sound incident normally onto the sample. In a practical environment sound is likely to be incident from all directions onto the absorbing material, and the normal-incidence absorption coefficient values do not give an accurate indication of the practical behaviour of the material under such conditions of random incidence. Making certain assumptions, a "statistical absorption coefficient" (α_{stat}) can be derived from the impedance tube measurements, which takes into account the random incidence of sound onto the material.

The literature surrounding the subject of sound absorption is very extensive, and one purpose of this Report is to provide a summary of the theoretical treatment of sound absorption. This summary is based on Kuttruff's published work.[1] The theoretical treatment leads to the derivation of formulae which can be used to calculate absorption coefficients from impedance tube measurements. Although both normal-incidence and statistical absorption coefficients are in fact calculated from the acoustic impedance of the sample, as measured in the impedance tube, they are not interrelated by a simple explicit formula. A graphical presentation of this relationship is therefore derived in this Report. The resulting chart can also be used to find values of the statistical absorption coefficient from measurements made using the impedance tube technique.

2. Normal-incidence absorption coefficient

A normal-incidence reflection coefficient (R_N) may be defined which expresses the change in amplitude and phase taking place during the reflection of a normally-incident plane sound wave from a surface. Since both amplitude and phase are involved, R_N is in general complex and thus

$$\left.\begin{aligned} R_N &= |R_N|e^{j\gamma} \\ &= |R_N|(\cos\gamma + j\sin\gamma) \\ &= R_R + jR_I \end{aligned}\right\} \quad \ldots(1)$$

where $|R_N|$ and γ are the modulus and argument of R_N, and R_R and R_I its real and imaginary parts.

Consider a plane wave-train normally incident onto a surface. The angular frequency of the source exciting this wave-train is ω. At any point the instantaneous sound pressure p_i is given by

$$p_i = \hat{p}_i \cos(\omega t) \quad \ldots(2)$$

where $\hat{p}_i$ is the peak sound pressure. The rms sound pressure is therefore given by

$$(p_i)_{\text{rms}} = \frac{\hat{p}_i}{\sqrt{2}} \quad \ldots(3)$$

Note that, because the time average of a sinusoidal function is zero, the rms value is independent of the frequency or phase of the pressure waveform.

Now consider the wave reflected from the surface. The instantaneous sound pressure (p_r) in the reflected wave is given by

$$\begin{aligned} p_r &= R_N p_i \\ &= p_i|R_N|e^{j\gamma} \\ &= |R_N|\hat{p}_i \cos\omega t . e^{j\gamma} \quad \ldots(4) \\ &= |R_N|\hat{p}_i \cos(\omega t + \gamma) \end{aligned}$$

Interference will take place between the incident and reflected waves. At some points the incident and reflected waves will be in phase, and the two wave-trains will reinforce each other to the greatest extent. The instantaneous sound pressure at these points ($p_{\max}$) will be the arithmetic sum of the sound pressures in the two components and will be given by

$$p_{\max} = \hat{p}_i(1 + |R_N|)$$

The rms sound pressure $(p_{\text{max}})_{\text{rms}}$ is therefore given by

$$(p_{\text{max}})_{\text{rms}} = \frac{\hat{p}_{\text{i}}(1 + |R_{\text{N}}|)}{\sqrt{2}} \qquad \ldots (5)$$

Similarly, at other points the two wave-trains will partially cancel, the rms sound pressure $(p_{\text{min}})_{\text{rms}}$ where the wave-trains are in antiphase and therefore cancel to the greatest extent being given by

$$(p_{\text{min}})_{\text{rms}} = \frac{\hat{p}_{\text{i}}(1 - |R_{\text{N}}|)}{\sqrt{2}} \qquad \ldots (6)$$

Thus the ratio S of maximum to minimum rms sound pressure is given by

$$S = \frac{(p_{\text{max}})_{\text{rms}}}{(p_{\text{min}})_{\text{rms}}} = \frac{1 + |R_{\text{N}}|}{1 - |R_{\text{N}}|} \qquad \ldots (7)$$

Equation 7 can be rearranged to give $|R_{\text{N}}|$ in terms of S, thus:

$$|R_{\text{N}}| = \frac{S - 1}{S + 1} \qquad \ldots (8)$$

If both the instantaneous sound pressure p_{i} and the instantaneous air velocity v_{i} of the incident wave are considered, then the sound power (or intensity) I_{i} is given by

$$I_{\text{i}} = \overline{p_{\text{i}} v_{\text{i}}} \qquad \ldots (9)$$

where the bar on the right-hand side of the equation indicates a time average. Further since in general[1b]

$$\frac{p}{v} = \rho_0 c \qquad \ldots (10)$$

where ρ_0 and c are the density of air and the velocity of sound in air respectively, Equation 9 can be written as

$$I_{\text{i}} = \frac{\overline{p_{\text{i}}^2}}{\rho_0 c} \qquad \ldots (11)$$

the bar on the right-hand side of the equation indicating time-averaging as before.

From Equations 2 and 11, and remembering that in general $\overline{\cos^2(k\omega t)} = \frac{1}{2}$ where k is any constant, it can be seen that

$$I_{\text{i}} = \frac{\hat{p}_{\text{i}}^2}{2\rho_0 c} \qquad \ldots (12)$$

The intensity (I_{r}) of the wave reflected from the surface is given, by analogy with Equation 11, by

$$I_{\text{r}} = \frac{\overline{p_r^2}}{\rho_0 c} \qquad \ldots (13)$$

Inserting the value of p_{r} given by Equation 4 and following through the argument leading to the derivation of Equation 12 gives the result

$$I_{\text{r}} = \frac{|R_{\text{N}}|^2 \hat{p}_{\text{i}}^2}{2\rho_0 c} \qquad \ldots (14)$$

Hence the ratio of the intensities of the incident and reflected waves is given by

$$\frac{I_{\text{r}}}{I_{\text{i}}} = |R_{\text{N}}|^2 \qquad \ldots (15)$$

The normal-incidence absorption coefficient α_{N} may now be defined as

$$\alpha_{\text{N}} = 1 - \frac{I_{\text{r}}}{I_{\text{i}}} \qquad \ldots (16)$$

$$= 1 - |R_{\text{N}}|^2 \text{ from Equation 15} \qquad \ldots (17)$$

$$= 1 - \left(\frac{S - 1}{S + 1}\right)^2 \text{ from Equation 8} \qquad \ldots (18)$$

Equation 18 may be rearranged to give

$$\alpha_{\text{N}} = \frac{4S}{S^2 + 2S + 1} \qquad \ldots (19)$$

In general, sound waves encountered in enclosures treated with patches of absorbing material are often spherical rather than plane since the distance between the point where the sound is radiated and the point where it is observed may not be great enough for the wavefronts to have become even approximately plane. This situation is particularly true at low audio frequencies, where the dimensions of the patches of acoustic treatment (or indeed the wall surfaces between such patches) are small or comparable with the sound wavelength. Such areas will tend to act as point sources emitting spherical waves. Plane sound waves will however be propagated along a tube whose diameter is rather smaller (a factor of about two is usually quoted) than with the wavelength, and will be partially reflected by a sample of absorbing material placed normally to the axis of the tube at the opposite end to the sound source used to excite the wave-motion. The resulting sound field can be explored using a suitable probe microphone and the ratio of greatest to least sound power measured. Equation 19 can then be used to calculate α_N. This is the basis of the "impedance tube" method (discussed in greater detail in the following sections of this Report) which is frequently used to assess the absorption coefficient of materials (e.g. carpets) when it is inconvenient to obtain the large sample required for reverberation room measurements.

3. Acoustic impedance

The acoustic impedance of a surface is defined as the ratio of the sound pressure to the component of air velocity normal to the surface of the sample, both quantities being taken at this surface. If, as before, the instantaneous pressure and velocity at the surface due to the normally-incident wave are p_i and v_i, and the corresponding quantities due to the reflected wave are p_r and v_r, then

$$v_i = \frac{p_i}{\rho_0 c} \text{(from Equation 10)} \qquad \ldots(20)$$

$$p_r = R_N p_i \text{ (from Equation 4, first version)} \qquad \ldots(21)$$

$$v_r = -\frac{R_N p_i}{\rho_0 c} \text{(from Equations 10 and 21)} \qquad \ldots(22)$$

The negative sign on the right-hand side of Equation 22 occurs since the reflected and incident sound-waves are travelling in opposite directions. The overall pressure at the surface (p_t) is given using Equation 21 by

$$p_t = p_i(1 + R_N) \qquad \ldots(23)$$

while the overall velocity (v_t) becomes, from Equations 20 and 22

$$v_t = \frac{p_i}{\rho_0 c}(1 - R_N) \qquad \ldots(24)$$

The acoustic impedance of the surface (Z) is then, from Equations 23 and 24

$$Z = \frac{p_t}{v_t} = \rho_0 c \frac{1 + R_N}{1 - R_N} \qquad \ldots(25)$$

Now Equation 10 can be interpreted as the characteristic impedance of "free air" (i.e. of a plane sound wave propagating in the absence of any obstructions), and the impedance of the surface can therefore be expressed in normalised form in terms of this quantity. Thus the normalised imedance or "specific acoustic impedance" ζ is given by

$$\zeta = \frac{Z}{\rho_0 c} = \frac{1 + R_N}{1 - R_N} \qquad \ldots(26)$$

and conversely

$$R_N = \frac{\zeta - 1}{\zeta + 1} \qquad \ldots(27)$$

The normal-incidence absorption coefficient may be found from Equations 17 and 27: thus

$$\alpha_N = 1 - \left|\frac{\zeta - 1}{\zeta + 1}\right|^2 \qquad \ldots(28)$$

Equation 28 may be expanded by using the relationship $|A|^2 = |A||A|^*$ where A is a complex quantity and A^* is its conjugate. Thus, denoting the real and imaginary parts of ζ by ζ_R and ζ_I,

$$\alpha_N = 1 - \left|\frac{\zeta_R - 1 + j\zeta_I}{\zeta_R + 1 + j\zeta_I}\right|^2$$

$$= 1 - \frac{|\zeta_R - 1 + j\zeta_I||\zeta_R - 1 - j\zeta_I|}{|\zeta_R + 1 + j\zeta_I||\zeta_R + 1 - j\zeta_I|}$$

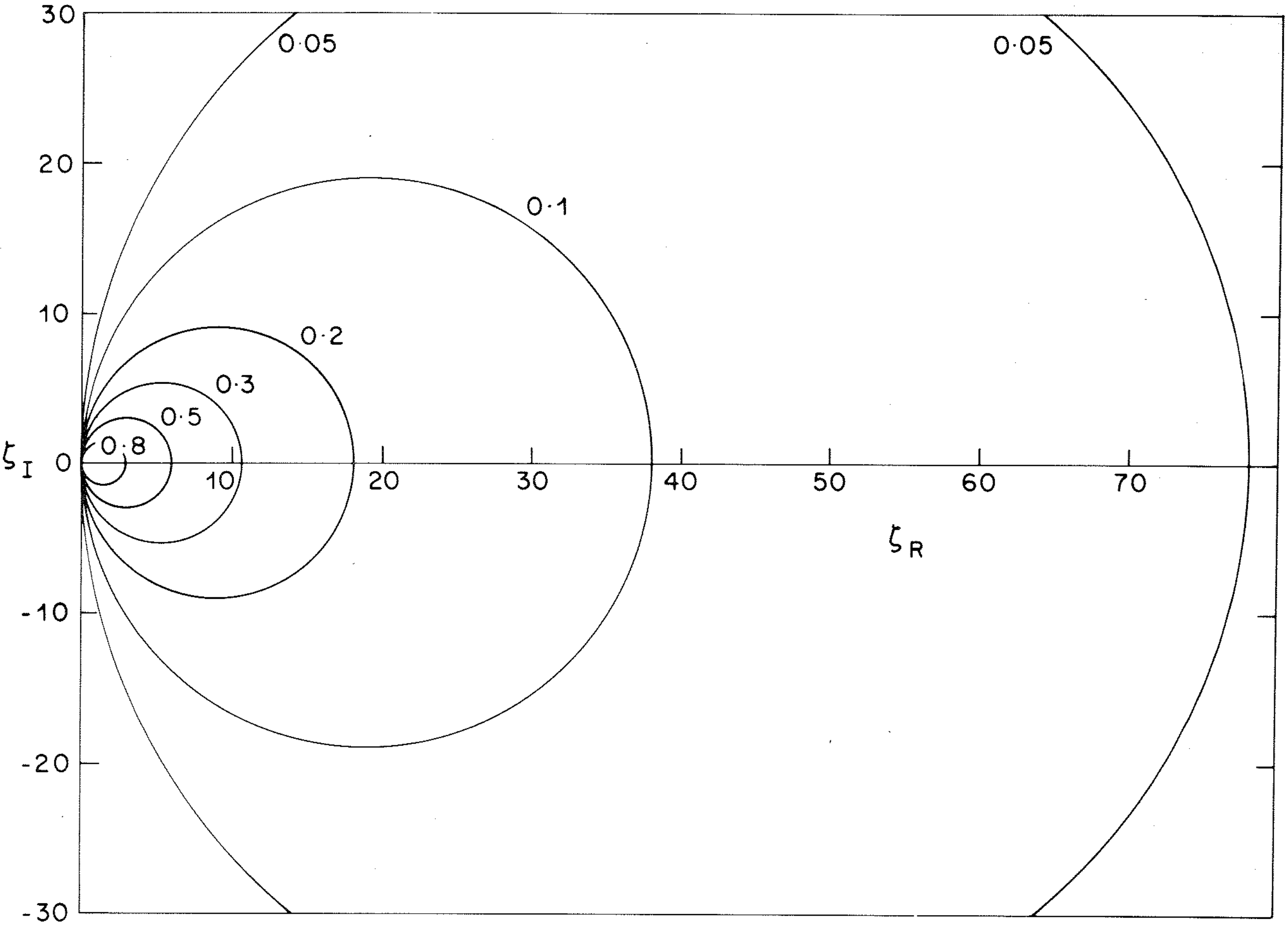
$$= 1 - \frac{(\zeta_R - 1)^2 + \zeta_I^2}{(\zeta_R + 1)^2 + \zeta_I^2}$$

By multiplying out this relationship, the following result is obtained:

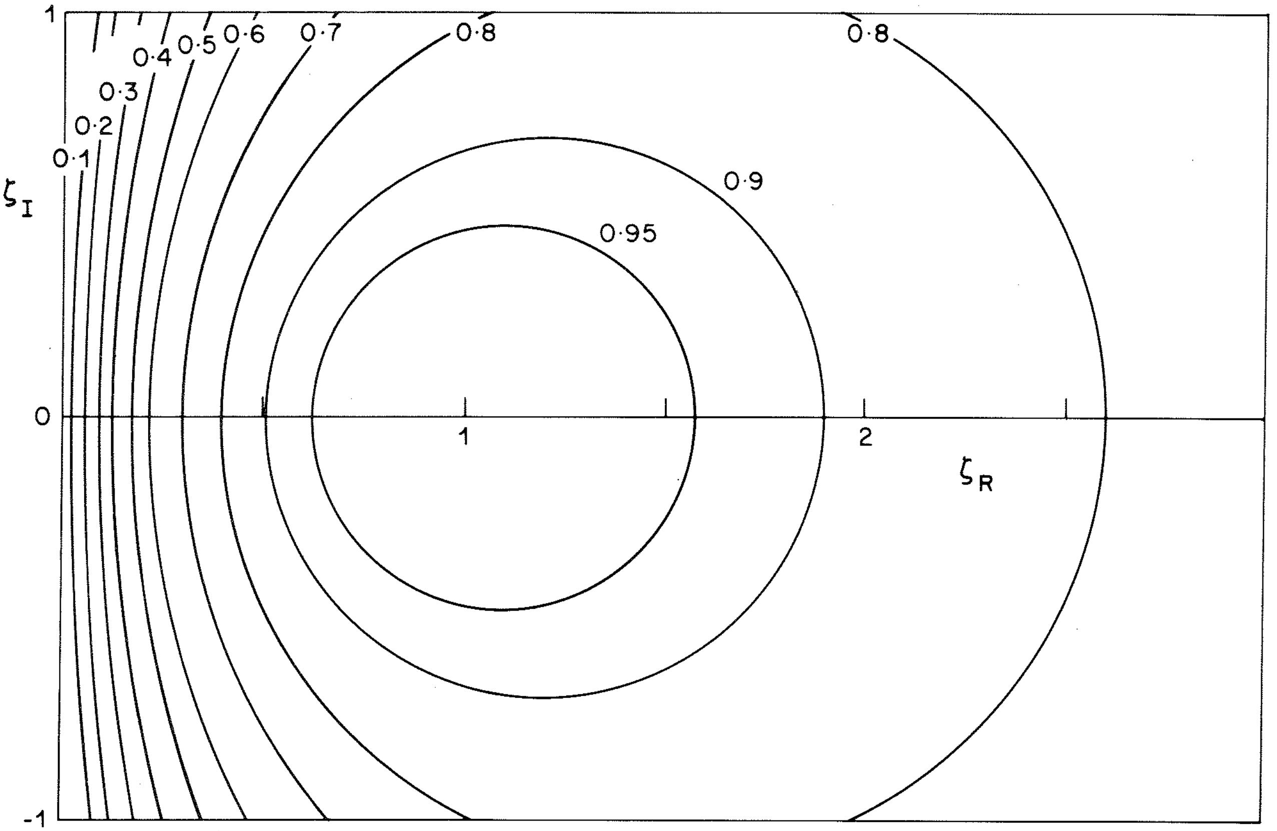
$$\alpha_N = \frac{4\zeta_R}{\zeta^2 + 2\zeta_R + 1} \qquad \ldots(29)$$

In Equation 29 $|\zeta|$ is the modulus of the specific acoustic impedance. By rearranging Equation 29 it can be shown that the loci of constant normal-incidence absorption coefficient in the complex ζ-plane[1c, 2b] (Fig. 1) are circles with centre $((2 - \alpha_N)/\alpha_N, 0)$ and radius $\sqrt{[(2 - \alpha_N)/\alpha_N]^2 - 1}$.

The value of the reflection coefficient R (see Equation 1) can be determined using the impedance tube as described in Section 2, by taking account of the distance (x_{min}) of the first sound pressure minimum from the absorbing surface, as well as the ratio of maximum to minimum sound pressure levels. The sound wave undergoes a phase shift of $360x_{min}/\lambda$ degrees, where λ is the sound wavelength, in traversing the path from the minimum to the point of reflection, and an equal phase shift in returning to the position of the minimum. The sound also undergoes a phase shift γ on reflection.


0·05
0·05
0·1
0·2
0·3
0·5
0·8
30
20
10
0
-10
-20
-30
ζ_I
10
20
30
40
50
60
70
ζ_R

(a)


0·1
0·2
0·3
0·4
0·5
0·6
0·7
0·8
0·8
0·9
0·95
1
0
-1
ζ_I
1
2
ζ_R

(b)

At the position of the minimum the total phase shift is 180°: thus

$$2(360x_{\min}/\lambda) + \gamma = 180$$

or

$$\gamma = 180(1 - 4x_{\min}/\lambda)\,(\text{degrees}) \qquad \ldots(30)$$

The value of $|R_N|$ can be calculated from the ratio of maximum and minimum sound pressures (S) using Equation 8: thus the impedance ζ can then be calculated using expanded versions of Equation 26, remembering that $R_R = |R_N| \cos \gamma$:

$$\left.\begin{aligned} \zeta_R &= \frac{1 - |R_N|^2}{|R_N|^2 - 2R_R + 1} \\[2ex] \zeta_I &= \frac{2R_I}{|R_N|^2 - 2R_R + 1} \end{aligned}\right\} \qquad \ldots(31)$$

These equations are the basis of the "Smith Chart"[2a, 3] originally developed for solving radio-frequency transmission-line problems. In principle the value of α_N can then be calculated using Equation 29. In practice this procedure is unnecessarily complicated since α_N can be calculated directly from the ratio of maximum and minimum sound pressures using Equation 19. The preceeding discussion nevertheless outlines the fundamental relationship between acoustic impedance, reflection factor and normal-incidence absorption coefficient. In Section 4 it will be seen that a knowledge of the acoustic impedance is in fact required in order to calculate the "statistical absorption coefficient". Furthermore, Equations 29 and 31 can be used to calculate the normal-incidence absorption coefficient in cases where the impedance tube method is not used, such as when the reflection coefficient is measured by examining the reflection of sound pulses from a surface, or when the acoustic impedance of a surface is calculated directly from its mechanical properties.

4. Statistical or random-incidence absorption coefficient

In general terms the acoustic impedance of a surface may vary with frequency, with position on the surface and with angle of incidence of the sound wave upon the surface. Some surfaces fall into a class termed "locally reacting", in which the normal component of air velocity at the surface depends only on the sound pressure at the element of surface under consideration and not on the pressure at neighbouring surface elements. In the case of these surfaces their impedance is independent of the angle of incidence of the sound waves. This property is assumed in the discussion which follows, as is the property that the impedance is unchanged with position on the surface.

In the case of oblique incidence onto a locally reacting surface, the reflection coefficient is a function of the angle of incidence. If θ is the angle of incidence (i.e. the angle between the normals to the wave-front and to the surface) then it can be shown (in a manner analogous to the derivation of Equation 27) that

$$R_\theta = \frac{\zeta \cos \theta - 1}{\zeta \cos \theta + 1} \qquad \ldots(32)$$

where R_θ is the oblique-incidence reflection coefficient at angle of incidence θ. Since ζ is independent of θ (the "locally reacting" assumption), the reflection coefficient depends on the value of $\cos \theta$. If $|\zeta| > 1$, and ζ is real (i.e. the acoustic impedance is purely resistive) then no sound will be reflected at an angle for which $\zeta \cos \theta = 1$. If a reactive component of impedance is present the reflected power will pass through a minimum at this angle of incidence but not fall to zero. At grazing incidence ($\theta \to 90°$) the reflection coefficient will tend to unity: thus no absorption of sound occurs under these conditions.

The absorption coefficient at an angle of incidence of θ (α_θ) is obtained in a manner similar to the derivation of Equation 29 and can be shown to be

$$\alpha_\theta = \frac{4\zeta_R \cos \theta}{(|\zeta| \cos \theta)^2 + 2\zeta_R \cos \theta + 1} \qquad \ldots(33)$$

Because of the "locally reacting" assumption, the value of the acoustic impedance can be measured at normal incidence using the impedance tube method (see Section 3) and then applied using Equation 33 to calculate the absorption coefficient at a given angle of incidence. The dependence of the reflection coefficient on the angle of incidence gives rise to a corresponding dependence of α_θ on θ: thus α_θ rises to a maximum at some value of θ if $|\zeta| > 1$ (the

Fig. 1—Loci of constant normal-incidence absorption coefficient. (a) $|\zeta| > 1$. (b) $|\zeta| \sim 1$. α_N values shown against loci

maximum value being unity if ζ is real) and falls to zero for sound arriving at grazing incidence to the surface.

The statistical absorption coefficient (α_{stat}) is obtained by making the assumption that it is equally probable for sound to arrive at a surface from any direction in front of the surface. Equation 33 can then be integrated over the entire hemisphere in front of the surface to arrive at the average or "statistical" value of absorption coefficient. This integration takes account of the fact that the effective area of the surface, for sound arriving at an angle of incidence θ, is diminished by a factor $\cos\theta$. The result of this integration is often expressed[1d, 4] in terms of the modulus and argument (β) of the specific acoustic impedance of the surface:

$$\alpha_{\text{stat}} = \frac{8}{|\zeta|}\cos\beta\left[1 + \frac{\cos 2\beta}{\sin\beta}\tan^{-1}\left(\frac{|\zeta|\sin\beta}{1+|\zeta|\cos\beta}\right) - \frac{\cos\beta}{|\zeta|}\log_e\left(1 + 2|\zeta|\cos\beta + |\zeta|^2\right)\right] \quad \ldots(34)$$

This relationship can also be written in terms of the real and imaginary parts (ζ_R and ζ_I) of the specific impedance of the surface:

$$\alpha_{\text{stat}} = \frac{8\zeta_R}{|\zeta|^2}\left[1 + \frac{\zeta_R^2 - \zeta_I^2}{\zeta_I|\zeta|}\tan^{-1}\left(\frac{\zeta_I}{1+\zeta_R}\right) - \frac{\zeta_R}{|\zeta|^2}\log_e\left(1 + 2\zeta_R + |\zeta|^2\right)\right] \quad \ldots(35)$$

Morse and Bolt have produced a chart[1e, 2c, 4] showing contours of equal α_{stat} plotted using co-ordinates of β and $|\zeta|$. (An alternative graphical method of evaluating α_{stat} from impedance tube measurements is given in Section 5 of this Report). In fact, α_{stat} may readily be calculated using a small computer, and this method is to be preferred from the point of view of both accuracy and convenience.

The fact that the absorption coefficient of a surface is zero for sound arriving at grazing incidence implies that the value of α_{stat} can never reach unity: in fact the maximum possible value of α_{stat} is 0.955, achieved when the specific acoustic impedance is real and equal to 1.6.

There is no reason, in principle, why the modulus of the oblique-incidence reflection coefficient of a surface ($|R_\theta|$) should not be measured at different angles of incidence θ, and corresponding values of absorption coefficient calculated directly, using Equation 17 (with $|R_\theta|$ substituted for $|R_N|$). An integration over the hemisphere in front of the surface could then be performed numerically to arrive at a quantity equivalent to the statistical absorption coefficient but not subject to the restriction of applying only to a locally-reacting surface. Davies and Mulhotland[5] have summarised the measurement techniques which can be used to measure the acoustic properties of surfaces at oblique incidence, and describe their own technique in which the change in waveform of an acoustic impulse on reflection is subjected to Fourier analysis.

5. Relationship between the normal-incidence and statistical absorption coefficients

The relationship between the normal-incidence absorption coefficient (α_N) and specific acoustic impedance (ζ) of a surface has been shown (Equation 29 and Fig. 1) to be such that loci of constant α_N are circles in the complex ζ-plane with centre $((2-\alpha_N)/\alpha_N,\ 0)$ and radius $\sqrt{[(2-\alpha_N)/\alpha_N]^2 - 1}$. Fig. 2 shows one such locus. The straight line locus (OD_1D_2) of values of impedance having argument β is also shown. The points D_1 and D_2 where this straight line intersects the circle of constant α_N represent conditions for which the given value of α_N is obtained when the argument of the impedance is equal to β. The lengths, OD_1 and OD_2 then give the values of $|\zeta|$ which satisfy these conditions. Using these values of $|\zeta|$ and β in Equation 34, the corresponding values of the statistical absorption coefficient (α_{stat}) can be calculated. It can be seen that, if the sign of β is changed, the same value of α_{stat} is obtained (the change in sign of the two $\sin\beta$ terms in Equation 34 compensate each other): consideration need only therefore be given to the upper half of the complex ζ-plane.

The values of $|\zeta|$ can be calculated by re-arranging Equation 29 so that the subject is $|\zeta|$:

$$|\zeta| = \frac{1}{\alpha_N}\left\{(2-\alpha_N)\cos\beta \pm \left[4(1-\alpha_N)\cos^2\beta - \alpha_N^2\sin^2\beta\right]^{1/2}\right\} \quad \ldots(36)$$

Here, again, the same value is (as would be expected from the geometry of Fig. 2) obtained if the sign but not the magnitude of β is changed. When the values of α_N and β are such that the term in square brackets is zero, the value of $|\zeta|$ corresponds to the length OD_3 in Fig. 2, which is obtained when $\beta = \beta_{\text{Lim}}$. For the given value of α_N, the argument of the specific acoustic impedance cannot exceed β_{Lim}. Since OD_3 is a tangent to the α_N-locus circle at D_3,

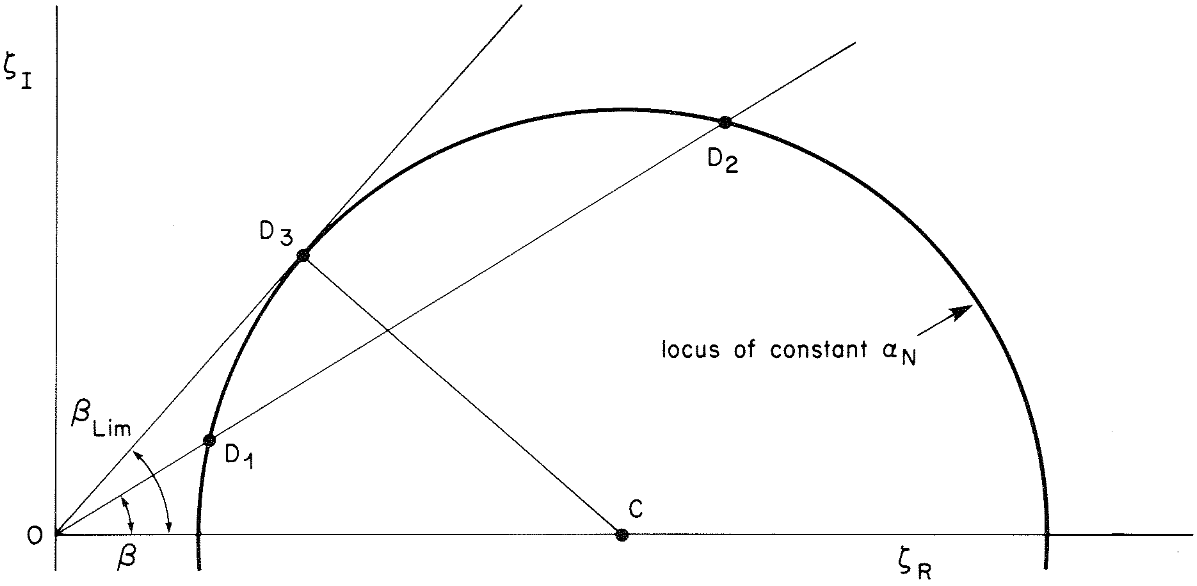


Fig. 2—Relationships between $|\zeta|$ *and* α_N. *The lengths OD, and* OD_2 *give values of* $|\zeta|$ *for given values of* α_N *and* β *(see text)*

$\angle OD_3C$ is a right-angle and it can be seen that

$$|\zeta|_{\text{Lim}} = \text{OD}_3 = (\text{OC}^2 - \text{CD}_3^2)^{1/2} = 1$$

where $|\zeta|_{\text{Lim}}$ is the value of $|\zeta|$ corresponding to the argument β_{Lim}, and OC and CD_3 take the values quoted at the beginning of this Section. Thus the locus of D_3 for different values of α_N is a circle with centre at the origin and unity radius. Since the radius of the locus of constant α_N decreases as α_N itself increases, it can be seen from Fig. 2 that the condition $|\zeta| = |\zeta|_{\text{Lim}} = 1$ corresponds to the highest value (α_{NLim}) of α_N which can be obtained for a given value of β. In physical terms, the ratio of sound pressure to normal air velocity at the surface is, under these conditions, the same as that obtained in "free air" (see Section 3), but a phase difference of β exists between them. It can also be seen that when α_N is less than α_{NLim}, and two solutions can be found for Equation 36, $|\zeta|$ will always be greater than unity in one case and less than unity in the other. The former condition is obtained when, for example, porous absorbent material is bonded directly onto a relatively massive surface. In this case the air velocity at the surface is less than would be obtained in free air for a given sound pressure. The same condition can also be obtained if the absorbent material is spaced from the backing surface by approximately an integral number of sound half-wavelengths (the exact spacing will determine the argument β as well as the value of $|\zeta|$). In such circumstances, however, the absorbing surface may not satisfy the "locally reacting" condition (see Section 4), unless precautions are taken to avoid the transverse propagation of sound in the space between the absorbing material and the backing surface. Values of $|\zeta|$ less than unity (where the air velocity at the surface is greater than would be obtained in free air for a given sound pressure) can be obtained when the distance between the porous absorbent material and the backing surface is approximately an odd multiple of a quarter-wavelength of the sound field.

Relationships between the normal-incidence and statistical absorption coefficients, obtained by the method outlined above, are shown in Fig. 3. The full lines refer to the relationships involving specific acoustic impedances greater than unity while the broken lines similarly involve impedances less then unity. The points for which $|\zeta|$ is equal to unity lie on the chain-dashed line. If the argument of the acoustic impedance (β) is not known (for example if α_N has been calculated from measurement of the ratio of the maximum and minimum sound pressure levels in the impedance tube, using Equation 19), then a given value of α_N may correspond to any value of α_{stat} lying within the α_N intercepts on the curve in Fig. 3 for $\beta = 0$. The construction of the absorber, particularly if it consists of porous material bonded to a massive surface, may indicate that $|\zeta|$ is greater than unity, and in this case the lower bound of the α_{stat} range would be given by the α_N intercept on the chain-dotted line rather than the lower (broken) line for $\beta = 0$.

As an example of the relationships between quantities measured using the impedance tube (the reflection factor, the specific acoustic impedance

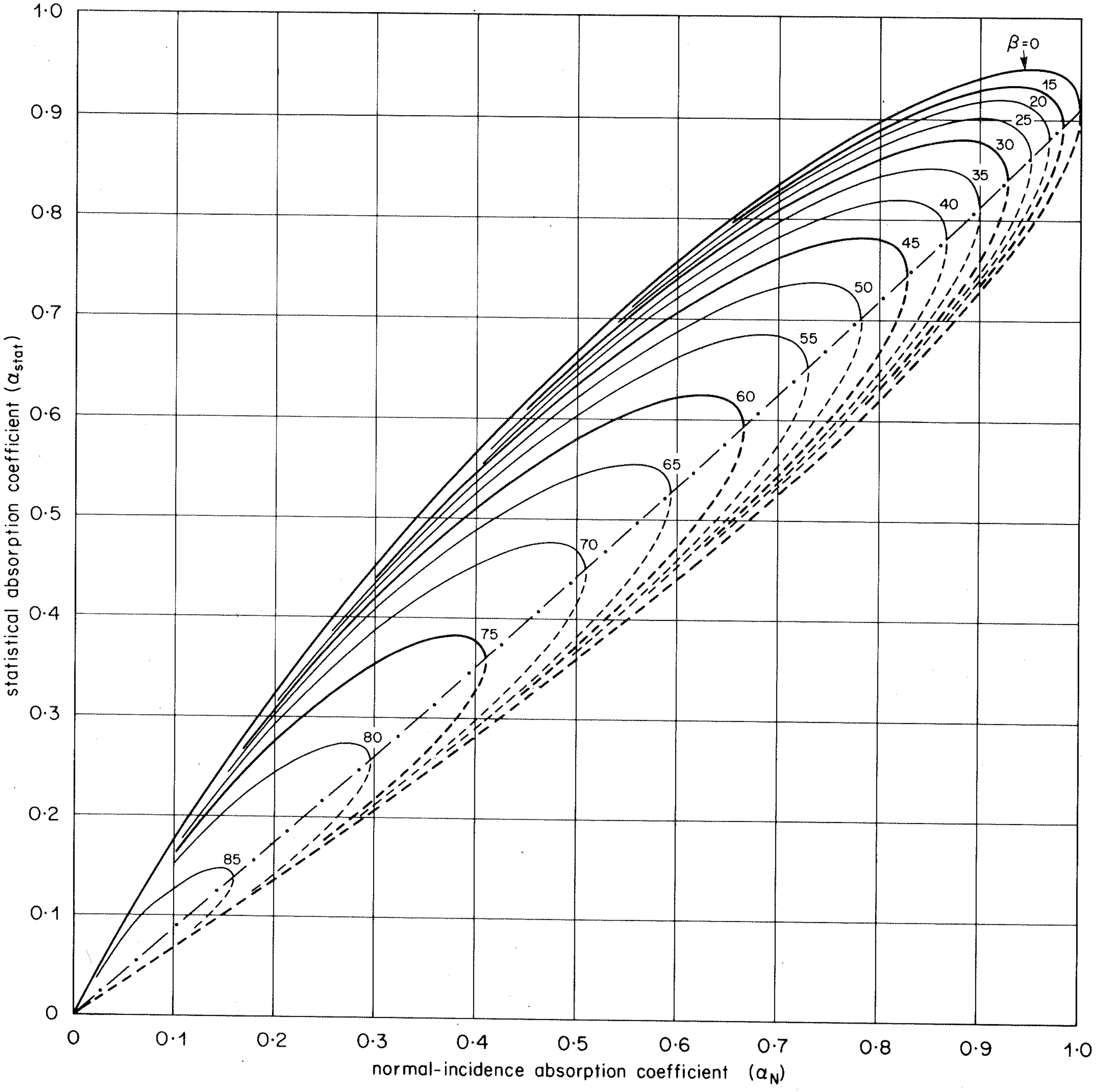


Fig. 3—Relationships between the normal-incidence and statistical absorption coefficients

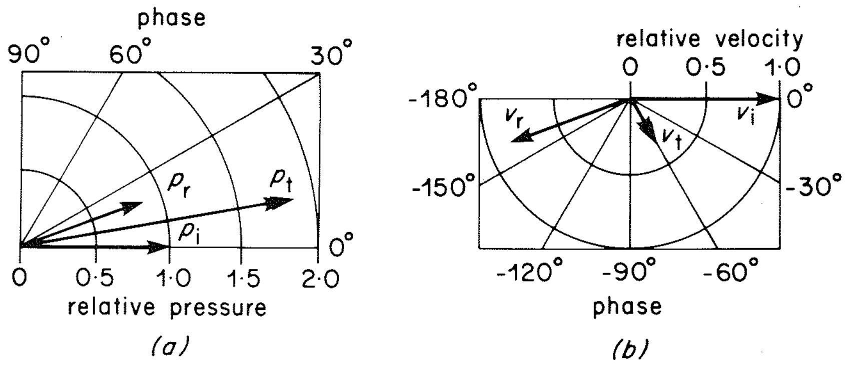


Fig. 4—Phasor diagrams of sound pressure and air velocity for sound wave reflected from a surface. (a) Sound pressure. (b) Air velocity

and the normal and statistical absorption coefficients) consider the measurement of a sample of carpet tile. At 2kHz the ratio of maximum to minimum sound pressure levels was 23.2dB. Application of Equation 8 gives a value for $|R_N|$ of 0.87, and application of Equation 17 (or Equation 19) shows that α_N is equal to 0.24. The position of the first minimum of sound pressure from the surface of the carpet tile (x_{min}) was found to be 38mm, and use of Equation 30 (inserting $\lambda = 172.3$mm) shows that γ is equal to 21°. Thus sound pressure and air velocity are both reduced by a factor of 0.87 and advanced in phase by 21° on reflection at the surface. These relationships are shown in vector or "phasor" form in Fig. 4: note that the air velocity undergoes a phase inversion in addition to the 21° shift, as shown in Equation 22. Assuming that the magnitudes of both the sound pressure and air velocity are unity in the incident sound wave, the resultant sound pressure in front of the surface is of magnitude 1.84 with a phase advance of 10° relative to the incident wave, while the air velocity in front of the surface is of magnitude 0.36 with a phase retardation of 59° relative to the incident wave. Fig. 5 shows the relationship between the resultant sound pressure and air velocity. The phasors have been redrawn in the conventional manner with the velocity vector (the acoustic analogue of electrical current) horizontal and equal to unity. The pressure phasor (the acoustic analogue of voltage) on this scale is of length 5.05 and leads the velocity vector by 69°. Using this convention, the length of the pressure phasor is numerically equal to the modulus of the specific acoustic impedance which thus takes the value 5.05. The resistive and reactive components of this impedance are $\zeta_R = 5.05 \cos 69° = 1.8$ and $\zeta_I = j.5.05 \sin 69° = j.4.7$ respectively. The equivalent circuit of the carpet tile at 2kHz is thus as shown in Fig. 6. Application of Equation 34 (or 35) shows that α_{stat} is equal to 0.34. This value of α_{stat} may also be deduced by inspection of Fig. 3, remembering $\alpha_N = 0.24$ and $\beta = 69°$.

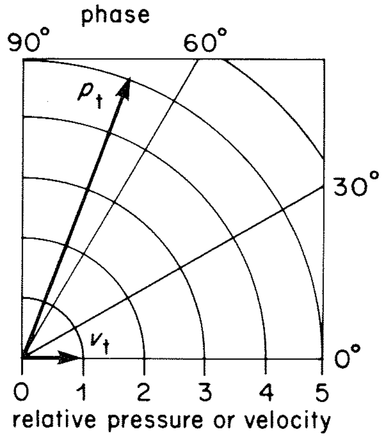


Fig. 5—Phasor diagram of resultant sound pressure and air velocity for reflection of sound at a surface

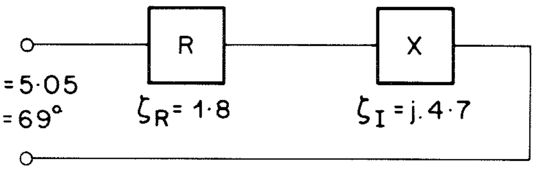


Fig. 6—Specific acoustic impedance of a carpet tile sample at a frequency of 2kHz

An important point to note is that the reactive component of the impedance is considerably greater than the resistive component, even though the displacement of the position of minimum sound pressure from a position one-quarter of a wavelength from the surface of the carpet tile is at first sight rather small (5mm). In fact, this consideration can be a source of uncertainty in the calculation of the resistive and reactive components of the specific acoustic impedance of materials such as carpet, since it may not be possible to define the position of the rather irregular and easily compressible surface to the order of accuracy required for precise calculations. Since the statistical absorption coefficient depends on ζ_R as well as $|\zeta|$ (see Equations 34 and 35) the value obtained for α_{stat} will be subject to a corresponding lack of precision. Under these circumstances it may be possible to estimate upper and lower limits to the value of x_{min} and thus derive corresponding limiting values of the specific acoustic impedance and of the value of α_{stat}. Alternatively, the position of the minimum of sound pressure level may be ignored and Fig. 3 used to define the range of possible α_{stat} values corresponding to the α_N value calculated using Equation 19. In the example given above, reference to Fig. 3 would indicate that a value of α_N of 0.24 corresponded to a range of α_{stat} values between 0.16 and 0.38, or possibly 0.22 to 0.38 if the construction of the absorber was considered.

Although the above example has been worked through in terms of phasor diagrams, it is clearly more convenient in practice to use Equations 31 in order to calculate the specific acoustic impedance, and then use Equation 35 (or alternatively the Morse and Bolt chart[1e, 2c, 4]) to derive α_{stat}. In fact, the relationships between α_N and α_{stat} shown in Fig. 3 may also be used to derive α_{stat} from the measurements of the maximum to minimum sound pressure level and the position of the first sound pressure minimum away from the absorbing surface, taken in an impedance tube. The procedure is as follows:

1. Calculate

$$|R_N| = \frac{S-1}{S+1}$$

where S is the ratio of the maximum and minimum sound pressure levels (see Equation 8).

2. Calculate

$$\gamma = 180\left(1 - 4\frac{x_{min}}{\lambda}\right)$$

where x_{min} is the distance of the first position of minimum sound pressure away from the surface of the absorber and λ is the wavelength of sound (see Equation 30: the value of γ is obtained in degrees).

3. Calculate $R_I = |R_N| \sin\gamma$.

4. Calculate

$$\beta = \tan^{-1}\frac{2R_I}{1 - R_N^2}$$

(see Equations 31).

5. Calculate

$$\alpha_N = \frac{4S}{S^2 + 2S + 1}$$

(see Equation 19).

6. Find α_{stat} from Fig. 3 using known values of α_N and β. Use the upper (full) lines if the value of γ (Step 2) is less than 90°, and the lower (broken) lines if γ is greater than 90°.

Note that $\lambda = c/f$ where c is the velocity of sound and f is the frequency of the excitation signal: furthermore $c = 331.45[(T + 273)/273]^{1/2}$ m/s, where T is the temperature in °C. As has previously been mentioned, the value of x_{min} requires measurement with some precision: it can be seen that the value of β obtained using Steps 2, 3 and 4 depends strongly on this measurement.

6. Conclusions

The "normal-incidence absorption coefficient" (α_N) describes the absorbing properties of a surface when plane sound waves fall on it at normal incidence. Since this condition is not often met with in practice, the "statistical absorption coefficient" (α_{stat}) has been derived. Making certain assumptions about the mechanism of absorption and the environment in which an absorber is placed, this coefficient represents more closely the practical behaviour of an absorber.

Using the impedance tube technique the reflection of sound at normal incidence from an absorber may be studied and the parameters of the complex "normal-incidence reflection factor" may be calculated. The modulus of the reflection factor, which may be obtained from measurement of the ratio of maximum to minimum sound pressure in the tube, leads directly to the value of α_N. The calculation of α_{stat} is approached by first determining the "specific acoustic impedance" of the absorber. This quantity (which is again complex) can also be calculated knowing the reflection factor, but now both the modulus and argument of this factor are required. The latter quantity is obtained by measuring in the impedance tube the distance of the first sound pressure minimum from the surface of the absorber.

Since the value of the normal-incidence absorption coefficient depends only on the modulus of the reflection factor, while the value of the statistical absorption coefficient depends on both the modulus and argument of this quantity, it is evident that no simple "one-to-one" relationship can exist between the two coefficients. A relationship of this kind can, however, be derived if the argument of the specific acoustic impedance is assumed to remain constant. A family of curves can then be drawn (Fig. 3) each curve representing the relationship between α_N and α_{stat} for a given argument of the acoustic impedance of the absorber. If only the value of α_N is known (for example, from a previous measurement), then the range of corresponding α_{stat} values may be found from this diagram. If the argument of the specific acoustic impedance is also known, then the corresponding α_{stat} value may be found precisely from the curves. In fact, the curves may be used as an alternative to the chart originally produced by Morse and Bolt[1e, 2c, 4] as a means of determining the statistical absorption coefficient from impedance tube measurements.

7. References

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    1(b) Ibid, p. 5.
    1(c) Ibid, Fig. II.2, p. 26.
    1(d) Ibid, Equation II.38, p. 44.
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